Constant multiple rule

$$\frac{d}{dx}\left(c \cdot f(x)\right) = c \cdot \frac{d}{dx}\left(f(x)\right)$$

Prove it. And explain the geometric intuition.

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Proof:

$$\frac{d}{dx}\left(c \cdot f(x)\right)$$

$$= \lim_{\Delta x \to 0} \frac{cf(x+\Delta x) - cf(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} c\left(\frac{f(x+\Delta x) - f(x)}{\Delta x}\right)$$

$$= c \cdot \frac{d}{dx}\left(f(x)\right)$$

Geometric explanation:

The derivative is the slope of the tangent line at that point.

The constant c stretches the function $f(x)$ in the y-axis, which also increases the slope by the constant c.